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A Theory of Solar Filaments

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Abstract

A theory of the stationary support of solar filaments (prominences) by the magnetic field of Babcock's field patches is proposed. Only at those places where the magnetic field is horizontal can a filament be supported. In addition conditions of stability have to be fulfilled. Some simple two-dimensional models of this kind are considered, and necessary stability conditions corresponding to a lateral displacement of the filament as a whole are derived. The theoretically derived structure and density of the filaments is found in satisfactory agreement with observations, only the thickness resulting being smaller than the usually quoted value, which, however, represents only an upper limit. Some remarks are made concerning a theory of the sudden disappearances ("catapult-theory") and this theory is compared with the theory of corpuscular emission ("melon seed theory") proposed earlier.

Note by Translator: Equation numbers referred to in the text agree with those in the original paper and are located to the right of the equation in question in the translation. Equation numbers to the left of the equations were added by the translator.

Introduction and Survey

In the following considerations of a theory of filaments we will limit ourselves to the stationary case. We ask, therefore, nothing as regards the transient development of the filaments, as regards their motions in total or in their inner motions. Considering the longevity of many large filaments there seems to be a legitimate question, whose answer may form the basis for later consideration on a theory of the life history of filaments.

A theory of filaments (or of stationary prominences) must answer, before all else, the question of the mechanism through which the three balances for the energy, for the mass flow and for the force become fulfilled. The question as regards the balance of energy bids others: how the relatively moderate temperature of the prominence comes about in its hot environment (namely the corona). A final answer will only be possible if the preliminary question as to the cause of the high temperature of the corona will be precisely and satisfactorily clarified. Qualitatively it is clear that the low temperature is closely connected with the large density (both relative to the surrounding corona) and that the energy loss by radiation in an optically thin layer with a constant temperature is proportional to the square of the density.

The conservation of mass of a filament is determined by the gain of material by the condensation of the falling coronal gases and by the loss by vaporization on the surface and finally by a possible mass flow into the sun's surface. If the theory of the supporting force doesn't allow too great a mass flow out from under the filament one expects no basic difficulty. We will come back to this later.

The question remains about the nature of the force which supports the filament against gravity and this question represents the essential subject of the preceding investigation. Heard in addition to this is an explanation for the characteristic ribbon-like structure of the fully formed filament - D'Azambuja (1948) states as typical dimensions for these: length 200,000 km, height 40,000 km, thickness less than 6,000 km.

Since neither gas pressure or radiation pressure are great enough to be sufficient in order to support a filament, and in addition give no explanation for the structure, it appears we have to employ magnetic forces. We will show that this assumption indeed corrects the balance of forces and unrestrictedly explains the leaf shaped or ribbon-like form.

A magnetic field $\vec{H}$ exerts a force on (non-polarized) material only when an electrical current $\vec{j}$ flows; the force density $\vec{k}$ is thereby given by

$$(1) \qquad \vec{k} = \frac{[\vec{j} \times \vec{H}]}{c} \qquad (1)$$

(c = velocity of light),

as well as having a value (neglecting the displacement current)

$$(2) \qquad c \vec{\nabla} \times \vec{H} = 4 \pi \vec{j} \,. \qquad (2)$$

In order to calculate the magnetic force on a filament one must know the magnetic field close by the stream in the filament and certainly must know the entire magnetic field inclusive of the sections produced by the flowing streams in the filament.

However, it is obvious how to eliminate the electric current and to consider the force directly as a result of the magnetic field:

$$(3) \qquad \vec{k} = - \frac{1}{4\pi} \vec{H} \times (\nabla \times \vec{H}) \,. \qquad (3)$$

This equation or the equivalent formulation with the help of Maxwell's stress tensor (which we will introduce in the following) allows the magnetic field lines to appear as the primary support and the force is then a sequence of the mutual repulsions of the field lines and the prolonged tension of the field lines. The advantage of this picture considering application to astrophysical problems lies in that because of the large dimensions and the sufficiently large conductivity, the movement of the material and the field lines are knotted closely with one another. However, then the electric current appears secondary, which follows from equation (2) because of the magnetic field will be deformed by the motion of the material.

The equations for the forces are also important if the conductivity is not infinitely large; since this is the only condition which we require of the electrodynamics, it is of considerable importance in the field configuration which we will consider in the following model for the support of filaments. The finiteness of the conductivity affects only the important non-static equilibrium where the matter moves slowly through the magnetic field in the direction of gravity. We will show, however, that this velocity is entirely unimportant (namely less than 1 mm/sec). A more extensive effect occurs because the magnetic forces affect primarily only the conducting material, that is the ionized part of the gas ("plasma"), while the neutral gas is linked to these only through friction and is diffused under the effect of gravity through the ionized components. This diffusion-velocity amounts to about 10 m/sec; which we will show is insignificant for the mass balance of a filament.

As evident from the picture of suspended matter in the elastically deformed field lines one can clearly compensate the force of gravity on the matter by a force from the magnetic field from above which will only be exerted where the magnetic field runs horizontally (that is parallel to the sun's surface). This follows formally from equation (3) or (1) because the force is always perpendicular to the direction of the field. The geometric position of all points on which the horizontal magnetic field lies forms a surface and if the structure of the magnetic field is somewhat simple, this surface will stand approximately perpendicular to the sun's surface. Henceforth the magnetic field is to be understood as that magnetic field distorted by the filament and certainly the field lines will be pulled from below through the filament. If the filament stands perpendicular the horizontal field will not run up to nor be displaced through the spot. The possible places where matter can be supported also lies on the surfaces of a structure which matches the observed filament.

We will treat the simplest model of a filament in the following two sections after this introduction. The simplifications are in the assumptions about the geometry, symmetry and independence of a horizontal coordinate and on the other side, in the neglect of the gas pressure in the filament where this becomes infinitely thin. The next most important question is if such a configuration is stable against small disturbances, because a position of unstable equilibrium can scarcely be adequate for a long-lived structure as the filament. The general stability problem with arbitrary disturbance is so difficult that we must again simplify considerably and begin with investigating the stability against an equal vertical and horizontal displacement. Presumably the latter is already among the perturbations which are independent of the horizontal coordinate.

Since the first model considered proved unstable we treat a somewhat more complicated case, in section 4, in which our stability criterion is fulfilled. Following this we consider the stability problem a bit more generally, whereby we expound that the incidents in the solar surface underneath the filament are particularly important.

In section 6 we bring up the negligence of the gas pressure in the filament and introduce in return additional geometric simplifications which appear justified by the preceding considerations. It gives as a result that the thickness of the filament is a small multiple of the scale height which delivers up the temperature in the filament.

In both of the following sections we discuss the resulting picture with an application on the sun and carry through a numerical estimate. Finally we discuss the possibilities of an empirical test of the proposed theory so as to find the relation of this theory to an earlier proposed mechanism for the acceleration of solar corpuscular radiation.

2. The Equilibrium Position of a Filament

The x,y-plane of a three dimensional coordinate system describes the sun's surface. We consider in the following (up to and including section 5) a filament as a thin layer of material with sufficiently large electrical conductivity (cp. section 6) in the y,z-plane above the solar surface. Assume that a magnetic field H is present in the half space belonging to the positive z axis whose sources may not be above the x,y plane but in the interior of the sun. The gravitational acceleration g exerts a directing force in the negative z direction on each position of the filament. The material in the filament is "hanging up" in the field lines of the magnetic field, because of its conductivity, therefore the filament can not fall along the z direction onto the solar surface. On the contrary it moves until the action of the force of the magnetic field compensates the gravitational force. The restrictions for this equilibrium condition must now be deduced.

One imagines that the filament is idealized by a infinitesimally thin layer of material in the yz plane.

(4) $f(y,z)$ [g cm^{-2}] should be the surface density of the material. The gravitational force on an element of area dydz of the filament is then gf dy dz. From the freedom of divergence of the magnetic field it follows that the x component H_x of $\vec{H}$ remains constant with the passage through the filament. If the magnetic field, however, should exert forces on the filament then H_z will sustain a discontinuity in the filament. The force density exerted by a magnetic field on matter is in general

$$(5) \qquad \vec{k} = - \operatorname{Div} \vec{T}, \ \vec{T} = (T_{ik}), \ T_{ik} = - \frac{1}{4\pi} (H_i H_k - \frac{1}{2} \delta_{ik} H^2) \qquad (3')$$

($i,k = x, y, z$; δ_{ik} is the Kronecker delta). With the aid of Gauss' Integral Theorem one obtains the vector for the force per unit area of the filament

$$(6) \qquad \vec{k} = (\vec{T}_1 - \vec{T}_2) \cdot \vec{r}$$

where r is the unit vector in the x direction, while T_1 and T_2 are defined by

$$(7) \qquad \vec{T}_1 = \lim_{x \to -0} \vec{T}, \qquad \vec{T}_2 = \lim_{x \to +0} \vec{T} \quad .$$

With the abbreviations

$$(8) \qquad \left\{H_x\right\} = H_{x1} + H_{x2} = \lim_{x \to -0} H_x + \lim_{x \to +0} H_x$$

$$(9) \qquad \left[H_x\right] = H_{x1} - H_{x2} = \lim_{x \to -0} H_x - \lim_{x \to +0} H_x$$

(y - and z - components correspondingly), one obtains with these the components of the magnetic force acting on the filament per cm^2

(10) $$K_x = \frac{1}{8\pi} \left(\left\{ H_y \right\} [H_y] + \left\{ H_z \right\} [H_z] \right) ,$$

(11) $$K_y = - \frac{1}{8\pi} \left\{ H_x \right\} [H_y] , \qquad (4)$$

(12) $$K_z = - \frac{1}{8\pi} \left\{ H_x \right\} [H_z] .$$

If the equilibrium in the z direction must prevail then

(13) $$\left\{ H_x \right\} [H_z] = - 8\pi \, g \, f . \qquad (5)$$

Should no force act on the filament in the x direction then

(14) $$\left\{ H_y \right\} [H_y] + \left\{ H_z \right\} [H_z] = 0 \qquad (5')$$

must be fulfilled in addition.

For the following we will employ preliminarily that all functions occurring are independent of y and that $H_y \equiv 0$. That means that the filament should occupy an infinite extension in the y direction and that the problem becomes two dimensional. $[H_y]$ then becomes automatically zero and employing (5') one has simply

(15) $$\left\{ H_z \right\} [H_z] = 0 . \qquad (6)$$

This condition is fulfilled when the sources of the field $\vec{H}$ lie symmetric to the plane $x = 0$ since then $\left\{ H_z \right\}$ vanishes on the basis of symmetry.

In order to mention an example of a magnetic field that supports a filament in equilibrium one must construct a magnetic field whose source does not lie over the xy plane and fulfills the restrictions of discontinuity (5), (5') respectively or in the two dimensional case (5), (6). This leads to a difficult theoretical potential problem. Essentially one finds the states of equilibrium of filaments in a magnetic field more easily if he first constructs fields which produce appropriate restrictions on the discontinuity and subsequently computes the distribution of material which stands in equilibrium with the previously constructed field.

As an example we consider the field of two fictitious magnetic monopoles which must sit along the straight line $x = \pm 1$, $z = 0$ and should possess different signs. The pole strengths along each constructed line are equal to a constant ± 1, so that the accompanying field is two dimensional. The field lines are then symmetric to the yz plane (Fig. 1a).

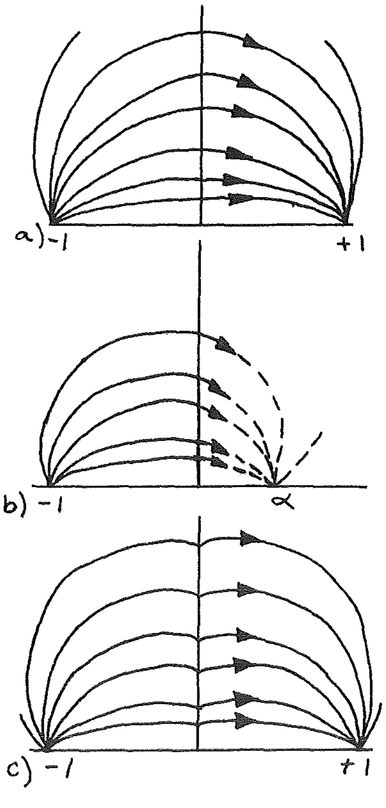

Fig. 1. Construction of a filament in equilibrium with its supporting field after treatment by reflection.

Consider the distribution on the straight line $x = +1$, $z = 0$ replaced by an equally valued constant distribution of the same sign along the line $x = \alpha$, $z = 0$, $0 < \alpha < 1$. The resulting field (Fig. 1b) is now no longer symmetric to the yz plane. Now we consider only the part of the field in region I ($x < 0$, $z > 0$) and imagine a new field in region II ($x > 0$, $z > 0$) produced by reflection of region I around the yz plane (Fig. 1c). The resulting total field is a potential field whose origin doesn't lie above the xy plane and that fulfills the restrictions on the origin in the yz plane.

In region I the field is measured by the potential

$$(16) \qquad \Phi_1 = \ln r_1 - \ln r_\alpha \; ; \qquad ((17) \qquad r_1^{\,2} = (x+1)^2 + z^2 \; ;$$

$$(18) \qquad r_\alpha^{\,2} = (x-\alpha)^2 + z^2 \;)$$

as a consequence of (19) $\vec{H} = +\,\mathrm{grad}\,\Phi_1$ we have:

$$(20) \qquad H_x = \frac{x+1}{r_1^{\,2}} - \frac{x-\alpha}{r_\alpha^{\,2}} \;, \qquad (21) \qquad H_z = z\left(\frac{1}{r_1^{\,2}} - \frac{1}{r_\alpha^{\,2}}\right).$$

In region II the potential of the reflected field is

$$(22) \qquad \Phi_2(x,\, z) = -\,\Phi_1\,(-x,\, z) \;,$$

thus

$$(23) \qquad H_x = -\left(\frac{x-1}{r_2^{\,2}} - \frac{x+\alpha}{r_{\alpha'}^{\,2}}\right), \qquad (24) \qquad H_z = -\,z\left(\frac{1}{r_2^{\,2}} - \frac{1}{r_{\alpha'}^{\,2}}\right)$$

with

$$(25) \qquad r_2^{\,2} = (x-1)^2 + z^2 \;, \qquad (26) \qquad r_{\alpha'}^{\,2} = (x+\alpha)^2 + z^2.$$

If one computes the function f from (5) with the constructed field one obtains the surface density of a filament that is forced by the magnetic field into the equilibrium condition shown in Fig. 1c. Condition (6) is thereby fulfilled on the basis of symmetry. One obtains for f

$$(27) \qquad f = \frac{z}{2\pi g}\left(\frac{1}{1+z^2} + \frac{\alpha}{\alpha^2+z^2}\right)\left(\frac{1}{1+z^2} - \frac{1}{\alpha^2+z^2}\right) .$$

The function f disappears at $z = 0$ and for $z \to \infty$, while for $z > 0$ it is positive.

A potential field $\vec{H}^o$ belongs uniquely to a magnetic field supporting a filament whose sources agree with that of $\vec{H}$, but which is continuous in the semi-infinite space along the positive z axis. Since $\vec{H}^o$ is the field that will occur if one eliminates the filament we will call $\vec{H}^o$ the undisturbed field. Thus the suitable undisturbed field in Fig. 1c is the field of Fig. 1a.

3. The Stability of a Filament

If a filament is maintained in equilibrium against gravity by a magnetic field, then the magnetic field deformed by the filament arises from the original (undisturbed) field H^o by superposing the magnetic fields H^f of the surface currents of the filament: $H = H^o + H^f$. The sources of the field H^o may lie symmetrical to the yz plane and not above the xy plane. Thus the symmetry conditions for H have the value:

$$(28) \qquad H_x^o\,(x,\ z) = H_x^o\,(-x,\ z), \qquad (29) \qquad H_z^o\,(x,\ z) = -\,H_z^o\,(-x,\ z) .$$

The symmetry of the field H^f further demands

$$(30) \qquad H_x^f\,(x,\ z) = H_x^f\,(-x,\ z), \qquad (31) \qquad H_z^f\,(x,\ z) = -\,H_z^f\,(-x,\ z) ,$$

thus the position of the filament has a value

$$(32) \qquad \left\{H_x^o\right\} = 2\,H_x^o , \qquad (33) \qquad [H_z^o] = 0, \qquad (34) \qquad \left\{H_x^f\right\} = 2\,H_x^f$$

(35) $[H_z^f] = 2\, H_{1z}^f$, (36) $(H_{1z}^f = \lim_{x \to 0} H_z^f)$, (37) $\left\{ H_z^f \right\} = 0$.

a) STABILITY AGAINST VERTICAL DISPLACEMENT. The z component of the magnetic force on the filament is by (4)

$$k_z = - \frac{1}{8\pi} \left\{ H_x \right\} [H_z] = - \frac{1}{2\pi} H_x H_{1z}^f \quad . \tag{38}$$

We now displace the filament in the z direction so that the matter which we found at the position z arrives at the position $z + \Delta z$. The displacement must not be necessarily fixed, that is Δz can be an arbitrary continuous function of z. Then the force which operates on a unit of surface area at the position $z + \Delta z$ is:

$$k_z + \Delta k_z = \frac{-1}{2\pi} H_x H_{1z}^f + \Delta k_z \tag{39}$$

$$= \frac{-1}{2\pi} (H_x H_{1z}^f + H_x \Delta H_{1z}^f + \Delta H_x H_{1z}^f) \quad . \tag{40}$$

The force of gravity fg per unit area is altered because of the displacement by $g \Delta f$ where Δf describes the change in surface density during the displacement. Since the field lines in the filament are "frozen solid" we can consider

$$\frac{\Delta f}{f} = \frac{\Delta H_x}{H_x} \quad . \tag{41}$$

With this the variation ΔK of the resulting force

$$K = k_z - fg \tag{42}$$

from the magnetic force and gravity, equals

$$\Delta K = \Delta k_z - g \Delta f = \Delta k_z - \frac{f g}{H_x} \Delta H_x \quad . \tag{43}$$

Which is by using (5)

$$\frac{f g}{H_x} = - \frac{1}{2\pi} H_{1z}^f \tag{44}$$

and it becomes

$$(45) \qquad \triangle K = -\frac{1}{2\pi} H_x \triangle H^f_{1z} \quad .$$

If we can now demonstrate that $H_x \triangle H^f_{1z}$ possesses the same sign as $\triangle z$, it follows that a resulting force acts on the filament (which was displaced $\triangle z$ in the z-direction) to drive back the filament to the position of rest. The relation to be proved

$$(46) \qquad \frac{H_x \triangle H^f_{1z}}{\triangle z} > 0$$

clearly means that the discontinuity which the field lines possess in the filament is made smoother by a displacement in the + z direction and more pointed by the displacement in the - z direction. In order to show this we drop, for a moment, the idealization of the infinitely thin filament and consider the equation in the interior of a filament of finite-thickness d (cp. Schlüter (1950), because of the neglect here cp. section 7)

$$(47) \qquad \frac{m_e}{e^2} \frac{\partial \vec{j}}{\partial t} = \frac{n_e}{c} [\vec{v} \times \vec{H}] - \frac{1}{ce} [\vec{j} \times \vec{H}] - \frac{n_e}{\sigma} \vec{j} + n_e \vec{E}$$

(m_e electron mass, e elementary charge, $\vec{j}$ current density, c velocity of light, $\vec{v}$ velocity of the plasma, n_e number of electrons per cm^3, σ ohmic conductivity, $\vec{E}$ electric field), whose y-component in the case

$$(48) \qquad v_x = v_y = 0, \; j_x = j_z = 0, \; H_y = H_z = 0$$

reads

$$(49) \qquad \frac{m_e}{e^2} \frac{\partial j_y}{\partial t} = \frac{n_e}{c} V_z H_x - \frac{n_e}{\sigma} j_y + n_e E_y \quad .$$

That we can set $H_z = 0$ is based on the fact that the field lines run essentially horizontal in the interior of the filament. On the basis of symmetry

$$(50) \qquad \int_{-d/2}^{d/2} H_z \, dx$$

vanishes.

In the preliminary considered case of infinite conductivity σ, the second term of the right side of the above mentioned equation equals zero and it follows for the stationary case $v_z = 0$. (An estimate for v_z in the case of finite conductivity will be given in section 7.) If we now displace the filament about $\Delta z = v_z \Delta t$, so that in the equilibrium position $E_y = 0$ (cp. section 7).

$$(51) \qquad \Delta j_y = \frac{e^2 n_e}{m_e c} H_x \Delta z.$$

We proceed with this result back to the infinitely thin filament and consider that because of

$$(52) \qquad \nabla \times \vec{H} = \frac{4\pi}{c} \vec{j} \, , \qquad (53) \qquad j_y^f = \lim_{d \to 0} (j_y d),$$

$$(54) \qquad n_e^f = \lim_{d \to 0} (n_e d)$$ (j_y^f = y component of the surface currents in the infinitely thin filament, n_e^f surface density of electrons)

$$(55) \qquad 2 H_{1z}^f = \left\{ H_z^f \right\} = \frac{4\pi}{c} j_y^f$$

so that it follows

$$(56) \qquad \Delta H_{1z}^f = \frac{2\pi}{c} \Delta j_y^f = \frac{2\pi}{c^2} \frac{e^2 n_e^f}{m_e} H_x \Delta z \, ,$$

thus

$$(57) \qquad \frac{H_x \Delta H_{1z}^f}{\Delta z} = \frac{2\pi}{c^2} \frac{e^2 n_e^f}{m_e} H_x^2 > 0 \, ;$$

thereby the stability is proved.

If we find a filament in the position of equilibrium, it is stable against vertical displacements.

b) STABILITY AGAINST HORIZONTAL DISPLACEMENTS. We already showed in section 2 that at the equilibrium position at each position of the filament k_x is zero because $\left\{ H_z^f \right\}_{x=o}$ vanishes. One now displaces the filament parallel to itself approximately a distance $\triangle x$ in the x direction, then at the new position k_x becomes

$$(58) \qquad k_x = \frac{1}{8\pi} \left\{H_z\right\}_{x=\triangle x} \left[H_z\right]_{x=\triangle x} ;$$

where we are again to take $\left\{H_z\right\}$, $\left[H_z\right]$ in the plane of the filament also in the plane of $x = \triangle x$. Also at this position

$$(59) \qquad \left\{H_z\right\} = \left\{H_z^o\right\} = 2 \, (H_z^o)_{x=\triangle x} ,$$

$$(59') \qquad \left[H_z\right] = \left[H_z^f\right] = 2 \, H_{1z}^f .$$

Only the original field $\vec{H}^o$ contributes to $\left\{H_z\right\}$ and only the field $\vec{H}^f$ contributes to $\left[H_z\right]$. Now

$$(60) \qquad (H_z^o)_{x + \triangle x} = \left(\frac{\partial H_z^o}{\partial x}\right)_{x = o} \triangle x ,$$

since in the plane $x = o$ H_z^o vanishes due to symmetry conditions and one obtains up to terms of higher order

$$(61) \qquad k_x = \frac{1}{2\pi} \left(\frac{\partial H_z^o}{\partial x}\right)_{x=o} H_{1z}^f \; \triangle x .$$

If $(\partial H_z^o/\partial x)_{x=o} \, H_{1z}^f < 0$ then the force k_x has the sign of $- \triangle x$, that is, it is exerting a force to push the displaced filament back again to its initial position; the position of the filament is stable. On the contrary, if $(\partial H_z^o/\partial x)_{x=o} \, H_{1z}^f > 0$ then the filament will be moved further away from the initial position by the force k_x; the position is unstable. Therewith the condition of stability is determined by the expression

$$(62) \qquad \left(\frac{\partial H_z^o}{\partial x}\right)_{x=o} H_{1z}^f .$$

Now by (4) at the position $x = \triangle x$

$$(63) \qquad 0 > -8\pi g f = \left\{H_x\right\} \left[H_z\right] = 4 \, (H_x^o) \; H_{1z}^f ,$$

and consequently $(H^o_x)_{x=\Delta x}\, H^f_{1z} < 0$. Therefore the signs of

$$\left(\frac{\partial H^o_z}{\partial x}\right)_{x=o} H^f_{1z} \quad \text{and} \quad - \left(\frac{\partial H^o_z}{\partial x}\right)_{x=o} (H^o_x)_{x=\Delta x}$$

are equal. Because $\nabla \times H^o = 0$ furthermore up to higher orders

$$(64) \qquad - \left(\frac{\partial H^o_z}{\partial x}\right)_{x=o} (H^o_x)_{x=\Delta x}\, \Delta x = - \left(\frac{\partial H^o_x}{\partial x}\right)_{x=o} (H^o_x)_{x=\Delta x}\, \Delta x$$

$$(65) \qquad = - \frac{1}{2} \left(\frac{\partial H^{o\,2}_x}{\partial z}\right)_{x=o} \Delta x \ .$$

Therewith it follows:

A FILAMENT IS STABLE AT HEIGHT Z AGAINST SIDEWARD DISPLACEMENTS WHEN

$$(66) \qquad \left(\frac{\partial H^o_z}{\partial x}\right)_{x=o} (H^o_x)_{x=o} = \frac{1}{2} \left(\frac{\partial H^{o\,2}_x}{\partial z}\right)_{x=o} > 0 \qquad (7)$$

IT IS UNSTABLE IN THIS POSITION IF THE FOREGOING EXPRESSION IS < 0.

We must define, in which sense the idea of stability is to be understood. We have displaced the filament entirely parallel to itself about Δ x. Thereby we have left the $\vec{H}^o$ part of the field unaltered and have made no assumptions about the field $\vec{H}^f$ that originates in the flowing streams in the filament. Outside of these we keep the filament in equilibrium at x = o. As stable we designate a position z of the filament where the matter in the field will be pushed back to the initial position after the displacement; as unstable, a place on which the field is forcing the displaced matter farther from the initial position.

As one regards the criterion, there is the capability of being able to support a filament in a stable equilibrium position which comes clearly from a condition of the undisturbed field H^o. (This condition is not trivial, for it was also conceivable, that a magnetic field was able to support a filament only if the filament possessed a definite distribution of matter f, or for example contained a certain minimum quantity of matter.) We will introduce in section 5 a generalization of the stability concept used here, for which generalization our criterion is no longer valid.

From criterion (7) it follows that the magnetic field H^o if it is able to support a filament stably at all it can also keep a filament of arbitrarily small total mass stably. From this the question naturally remains untouched, because of course the question is independent of whether the filaments of arbitrarily total mass can form at all on the sun. This is so because we previously neglected (the energy balance, and the mass balance of a filament) which are very likely essential for this.

From criterion (7) one can easily read off that a field H^o can exactly stably support its filament matter at the position of its plane of symmetry on which the field lines are curved above the surface. From this it follows directly that the example of the field between two magnetic monopoles, considered in section 2, is not in the position to support a stable filament. The thus constructed filaments are therefore unstable. In the next section we will give an example of a field that can support the stable filament.

4. The Field of Two Dipoles

On both straight lines $x = \pm 1$, $z = 0$ let two magnetic dipoles of opposite sign be so that the accompanying two dimensional magnetic field is given by the potential

$$(67) \qquad \Phi_o = - z \left(\frac{1}{r_1^{\,2}} - \frac{1}{r_2^{\,2}} \right) .$$

In figure 2 the field lines are produced and one sees immediately that they are curved concave above the surface in a certain region of the yz plane and that the field in this region can support a stable filament. In the results that follow

$$(68) \qquad H_x = \frac{2(x+1)z}{r_1^4} - \frac{2(x-1)z}{r_2^4}$$

also

$$(69) \qquad \frac{\partial H_x}{\partial z} = -\,[3z^2 - (x+1)^2]\,\frac{2(x+1)}{r_1^6} + [3z^2 - (x-1)^2]\,\frac{2(x-1)}{r_2^6}$$

for the region $x = o$ and for $z > o$

$$(70) \qquad (H_x)_{x=o} = \frac{4z}{r^4} > 0,$$

$$(71) \qquad \left(\frac{\partial H_x}{\partial z}\right)_{x=o} = \frac{4}{r^6}\,(1 - 3z^2), \quad (r = (r_1)_{x=o} = (r_2)_{x=o}) \ .$$

Also $\left(\dfrac{\partial H_x^2}{\partial z}\right) > o$ if $o < z < 1/\sqrt{3}$. Up to the height $z = 1/\sqrt{3}$ the field can support a stable filament. If one considers a filament of any division of matter f, brought into the field of figure 2a, the surface currents flowing in the filament evoke a magnetic field $\vec{H}^f$ so that the field $\vec{H}^o + \vec{H}^f$ at the bend in the plane of the filament maintains the suitable equilibrium conditions (5) and (6).

In analogy to the example of section 2, let us now construct a field by employing the method of reflection, so that one can regard it as an approximation to the field $\vec{H}^o + \vec{H}^f$. Since the method of reflection is a very special construction procedure let us only make use of the completely determined mass division of a filament. As in the earlier example, one must first work out the reflection and then pick off which division of matter belongs to the supporting field.

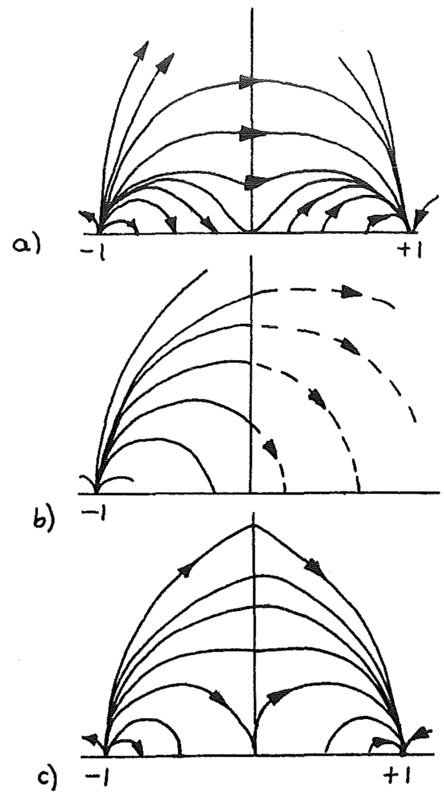

Fig. 2. The construction of a filament in stable equilibrium of a supporting field by the method of reflection.

The reflection procedure goes as before in that one takes a dipole belonging in the straight line x = -1, z = 0. The potential is given by

$$\Phi_1 = - \frac{z}{r_1^{\,2}} \quad . \tag{72}$$

The field lines are then circles (Fig. 2b). Now one considers the field in region II (x > o, z > o) and replaces it by the field one obtains if one reflects region I (x < 0, z > o) around the yz plane (Fig. 2c). The field lines thus obtained are then due to a potential

$$
(73) \quad \Phi = \begin{cases} \Phi_1 = -\dfrac{z}{r_1^2} \quad (x < o) \quad , \\[2em] \Phi_2 = \dfrac{z}{r_2^2} \quad (x > o) \ . \end{cases}
$$

The new field $\vec{H}$ possesses the same sources as the field $\vec{H}^o$ of fig. 2a. The field $\vec{H} - \vec{H}^o$ is therefore source free and possesses positions of discontinuities in the yz plane, which comply with the surface streams in the yz plane. These streams can associate with a filament whose division of matter f is suitable to determine the equilibrium condition (4). It gives as a result

$$
(74) \qquad f = \frac{1}{\pi g} \, \frac{1}{r^8} \, z\,(1 - z^2), \; r^2 = 1 + z^2 \quad . \qquad (8)
$$

One can also explain the field of figure 2c as the field that one obtains if one brings in a filament into the undisturbed field $\vec{H}$ of figure 2a whose division of matter is determined by (8).

The filament constructed by the simple reflection method shows some flaws. As we have seen above the undisturbed field can only support a stable filament up to a height of $z = 1/\sqrt{3}$. The filament constructed here has, to be sure, about 80% of its mass in the stable region. However, it also extends into the region $1/\sqrt{3} < z < 1$. The filament can also possess matter up there although it cannot contain stable matter. For $z > 1$ the function f even becomes negative since there the field of figure 2c exhibits breaks from above. Also our reflection procedure supplies no consistent solution. Since, however, f goes to zero as $1/z^5$ for great heights the physically meaningless negative densities die away and their effect on the field in the region $o < z < 1/\sqrt{3}$, in which we find the bulk of the filament, is not of importance. Thus one can regard the field constructed here as an approximation of a field supporting a stable filament.

5. A Generalization of the Stability Concepts

By the derivation of criterion (7) a filament becomes stable because after a displacement against the source of the field it is forced to return to its original position. The field $\vec{H}^o + \vec{H}^f$ alters itself in the entire volume by the displacement and only the sources remain unchanged. We must now consider displacements of the filament where the magnetic field strength along the entire solar surface remains unchanged. The essential difference compared with the earlier considerations will be made clear by the following example. In the two dimensional field of a two magnetic monopole distribution (Fig. 3a) assume a filament in an equilibrium position. As a consequence of the weight of the filament the field lines are squeezed underneath while more field lines may penetrate into the solar surface (c.p. Fig. 3b). One now asks about the stability of the filament. He now considers small sideward displacements of the filament from its equilibrium position. But only the field lines over the solar surface can easily follow the movement of the filament. Field lines which develop in part under the solar surface, as is shown in figure 3b by broken lines, become practically frozen with small motions of the filament because of the dense matter beneath the solar surface. Hence one can recognize directly the generalization of criterion (7); that instead of the field $\vec{H}^o$ which is kept constant there now comes the field $\vec{H}^1$ which is defined as that field which above the solar surface is a potential field and which at the solar surface agrees with that field which is present there if the filament is supported in an equilibrium position.

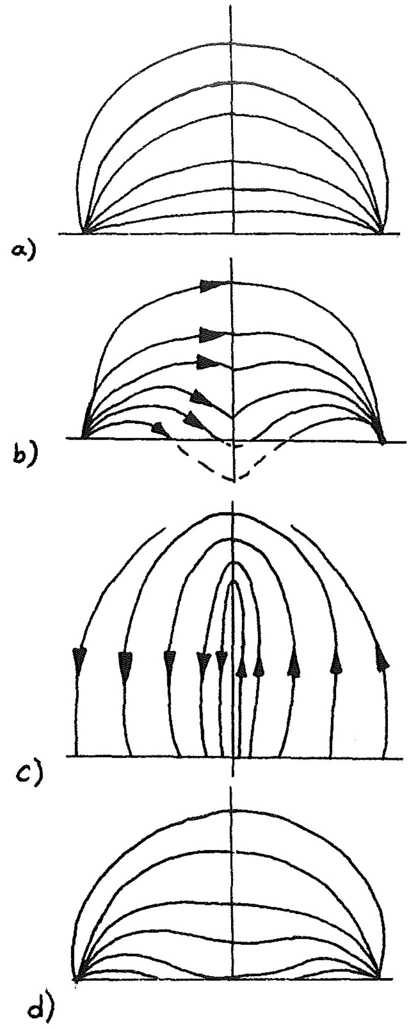

Fig. 3. Possible case of the penetration of field lines into the solar surface. a) undisturbed field $\vec{H}^o$, b) total field $\vec{H}$, c) field $\vec{H}^f$ of the streams in the filament, d) the field $\vec{H}^1 = \vec{H} - \vec{H}^f$.

Without the filament there is the field $\vec{H}^o$ above the xy plane; if a filament is taken into that field and if there is equilibrium then one can decompose the existing field $\vec{H}$ into

$$\vec{H} = \vec{H}^f + \vec{H}^1 \quad . \tag{75}$$

Here $\vec{H}^f$ is the field of the currents flowing in the filament (over the solar surface) (Fig. 3c). The field $\vec{H}^1$ possesses the same sources as $\vec{H}^o$.

The difference of $\vec{H}^o$ from the field $\vec{H}^1$ will be shown in our example of field lines which break through the solar surface several times. The field $\vec{H}^1$ now replaces the position of $\vec{H}^o$ in the stability criterion (7). It gives, therefore:

(76) We consider a filament in an equilibrium position in a magnetic field and $\vec{H}$ is the field strength of the total field (deformed by the filament) and $\vec{H}^f$ the field strength of the field caused by the flowing currents in the filament, so the filament is stable in altitude z when the lines of the field $\vec{H}^1 = \vec{H} - \vec{H}^f$ are concave above the surface. (9)

If $\vec{H}^1 = \vec{H}^o$, that is if the filament pushes no field lines under the solar surface, criterion (7) and (9) are identical. The direct result of our generalization is that a characteristic of the field $\vec{H}^o$ is that it is no longer able to support a stable filament; it depends more on the fact that the filament changes the undisturbed field $\vec{H}^o$. We consider once more the example of the two dimensional field of two magnetic monopole distributions. The filament changes the field $\vec{H}^o$ only so that the total field $\vec{H}$ doesn't push any field lines through the solar surface between the two poles. It follows from criterion (7) that there can be no stable filament. If on the other hand the filament presses field lines under the solar surface during its formation then $\vec{H}^1 \neq \vec{H}^o$ and criterion (9) means that in a certain region a filament may be supported stably (c.p. Fig. 3d).

We have discussed in this section merely the sequence for stability which the penetration of field lines into the solar surface brings about without, however, going farther into the mechanism of the penetration of the field lines. Certainly the penetration of a field $\vec{H}^o$, which according to criterion (7) is not able to support filaments, can only occur if the filament possesses a certain minimum mass. Filaments of arbitrary small mass will simply flow down along the field lines onto the solar surface.

It is necessary to show the circumstances which we have not considered up to now. Here, as in section 3, we have retained the fact that the current flowing in the filament is not changed when the filament becomes displaced by an amount Δ x. An alteration of the current can come about since the filaments distribution of matter f, changes with the displacement, while the number of field lines which penetrate the filament per cm^2 are altered by the displacement. Because of the large conductivity the flux through a deformed surface element moving with the matter remains constant. Then one must change f directly, so that

$$\text{(77)} \qquad \frac{\Delta H_x^o}{H_x^o} = \frac{\Delta f}{f} \qquad \text{(10)}$$

where ΔH_x^o, Δ f should be the changes at the position of the filament with the displacement of Δ x. Since on the basis of symmetry conditions

$$\text{(78)} \qquad \left(\frac{\partial H_x^o}{\partial x}\right)_{x=o} = 0 \ ,$$

condition (10) apart from magnitudes of higher order is automatically satisfied whereby our derivation is satisfied.

6. The Microstructure of a Filament

In contrast to the earlier simplifications we shall now consider the force of gas pressure in the filament and for that reason will assume that the filament possesses a finite thickness. If the matter becomes held by the magnetic field in equilibrium against gravity then at each place the equilibrium condition

$$(79) \qquad \vec{g}\,\rho - \text{grad } P - \frac{1}{4}\pi\,[\vec{H} \times (\nabla \times \vec{H})] = 0$$

holds, where $\vec{g}$ represents the vector of gravitational acceleration, ρ represents the density of matter, and P represents the pressure. One again retains the two dimensional problem considered earlier and obtains with the aid of the gas equation

$$(80) \qquad P = \frac{R}{\mu}\,\rho\,T$$

and the assumption of constant temperature

$$(81) \qquad -4\pi\,\frac{R}{\mu}\,T\,\frac{\partial \rho}{\partial x} - H_z\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) = 0\,, \qquad (11)$$

$$(82) \qquad -4\pi\,g\,\rho - 4\pi\,\frac{R}{\mu}\,T\,\frac{\partial \rho}{\partial z} + H_x\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) = 0. \qquad (12)$$

One differentiates (11) with respect to z and (12) with respect to x and subtracts both equations so that one has after consideration that $\nabla \cdot H = 0$

$$(83) \qquad 4\pi\,g\,\frac{\partial \rho}{\partial x} = H_x\,\Delta H_z - H_z\,\Delta H_x, \quad \left(\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right),$$

from which with (11) equation (13) follows

$$(84) \qquad H_x\,\Delta\,H_z - H_z\,\Delta\,H_x + \frac{H_z}{h}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) = 0 \qquad (13)$$

in which h is set equal to $\frac{RT}{g\mu}$.

The components of the field $\vec{H}$ must satisfy this differential equation if the filament is to be in equilibrium. However, the problem still permits a few simplifications. In a very thin filament H_x changes slightly with x so that nearby it appears that H_x is independent of x in the filament. We consider that H_z is only a function of x. Because $\nabla \cdot \vec{H} = 0$ one is allowed to make H_z not dependent on height z. We employ therefore that H_z is only a function of x. Further we assume that H_x is not dependent on z whereby H_x becomes simply a constant. The simplifications forced on the problem are obvious if one considers that in a filament H_z is very strongly dependent on x (at the border crossing to the infinitely thin filament H_z has a position of discontinuity!), while on the other hand the change of H_x in the filament is very small ($\partial H_x/\partial x$ vanishes in the middle of the filament on the basis of symmetry!).

With these simplifications (13) reduces to an ordinary differential equation for $H_z(x)$:

$$(85) \qquad \frac{\partial^2 H_z}{\partial x^2} + \alpha H_z \frac{\partial H_z}{\partial z} = 0, \quad \alpha = \frac{1}{h\, H_x} = \frac{g\mu}{RTH_x} \; . \qquad (14)$$

As boundary conditions at the position x = 0 it becomes required that $H_z = 0$ - a condition which again results on the basis of symmetry. For $x \rightarrow + \infty$ it should become required that H_z reaches an accessible value H_z^∞, for then $\frac{H_z^\infty}{H_x} > 0$. Therewith the solution of (14) is clearly determined. It is

$$(86) \qquad H_z = H_z^\infty \tanh \left(\frac{H_z^\infty}{H_x} \frac{x}{2h} \right) , \qquad (15)$$

where $h = \frac{RT}{g\mu}$ is the thickness of the homogeneous layer ("scale height"). One can easily convince himself that it satisfies the equation by putting this expression into (14). Because tanh(o) = o and $\lim_{x \rightarrow +\infty} \tanh(x) = 1$ it follows that it fulfills the desired boundary conditions.

For the calculation of the density distribution ρ (x,z) we consider equation (11) which has the form in our case

$$(87) \qquad 4\pi \frac{R}{\mu} T \frac{\partial \rho}{\partial x} = - \frac{1}{2} \frac{\partial H_z^2}{\partial x} \; .$$

One brings up the boundary condition $\lim_{x \rightarrow \infty} \rho = 0$, so the above equation becomes

$$(88) \qquad 4\pi \frac{R}{\mu} \rho\, T = -\frac{1}{2} (H_z^\infty)^2 \left[\tanh^2 \left(\frac{H_z^\infty}{H_x} \frac{x}{2h}\right) - 1\right] .$$

Therewith it is equivalent to

$$(89) \qquad 4\pi \frac{R}{\mu} \rho\, T = \frac{1}{2} (H_z^\infty)^2 \frac{1}{\cosh^2\left(\frac{H_z^\infty}{H_x} \frac{x}{2h}\right)} . \qquad (16)$$

In Figures 4 and 5 the behavior of the field lines and the behavior of the density are represented as functions of x. One observes directly from it that the order of magnitude of the diameter of a filament is 4 $(H_x/H^\infty{}_z)$ h.

The physical meaning of the solution is that we have a barometric density and pressure change along the field lines; from this follows the thickness if the slope of the field lines is known as the ratio H_x/H_z^∞.

If we here draw on equation (5) from section 2 (and identify H_z^∞ with the H_{z1} at the point), is with given H_x and given mass f per unit surface area (= $\int \rho\, dx$) fix H_z^∞ and for the thickness it follows

$$(90) \qquad d \approx \frac{2}{\pi} \frac{H_x^2}{g\, f} h .$$

We have here regarded the temperature as constant and neglected the gas pressure in the corona. If we assume that the rise of temperature in the lateral direction results in a thickness which is small, compared with the thickness of the filament (that is to say along a small mean free path) and that the pressure in the corona is comparatively constant then this amounts to a change of our boundary conditions. The best estimates say that the pressure in the filament is almost equally as large as the pressure in the undisturbed corona at the same height above the solar surface (Zanstra, 1947). This comes to

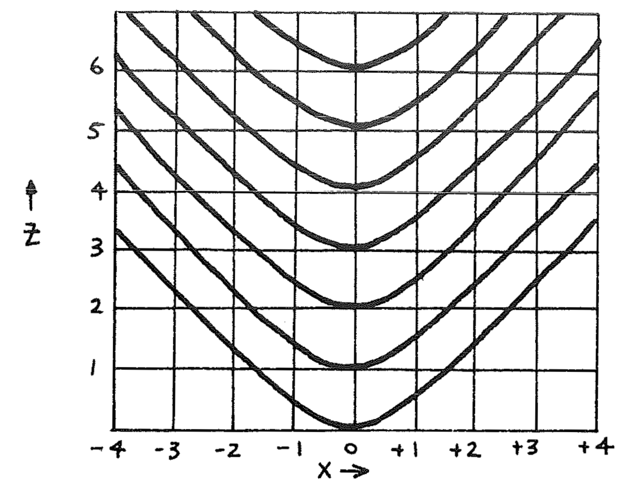

Fig. 4. Course of the field lines in a filament. Units on abscissa $\frac{2h\ H_x}{H_z^\infty}$, unit ordinate 2h.

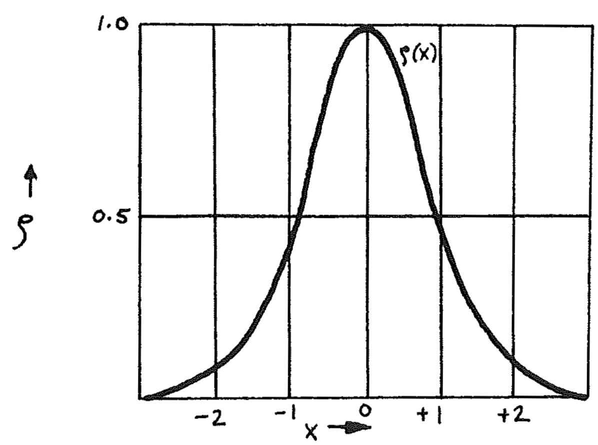

Fig. 5. Density distribution ρ(x) in a filament. Unit abscissa $\frac{2hHx}{H_z^\infty}$. The unit of ordinate is so chosen so that ρ(o) = 1.

disagreement with the theory presented here since pressure equilibrium should govern how the temperature rises and the pressure near the inside of the filament should grow barometrically along the field lines. In fact, however, the coronal density in the vicinity of a filament appears to be less than in the undisturbed corona. This is to be expected theoretically because of the condensation of mass through the filament ("cold trap effect").

The condensation follows along the field lines which intersperse the filament and the corona here. A relative dilution of the corona by a factor of 3 or 4 in contrast to Zanstra's hypothesis would leave our determination of the density of the filament practically unchanged.

7. The Maintenance of the Electric Current

Up to now we determined only the form of the magnetic field that is essential in order to retain the equilibrium of mechanical forces of the filament. We have therefore considered conditions that can exist in a stationary filament. These conditions are independent of the question about the cause of currents flowing in the filament. The electric current is prescribed by the condition of equilibrium of forces, the electric field, and the state of motion of the matter. These must be so placed that the required current flow is given directly by the conductivity equation. (This is not a necessity for the component of the electric current which flows parallel to the magnetic field since this exerts no force. We have rejected such components so far through our symmetry assumptions.)

The conductivity equation under the adopted negligences (especially the cause of the gradients of the partial pressure and of the neutral gases) (Schlüter, 1950), is

$$(91) \qquad \vec{E} + \left[\frac{\vec{V}}{c} \times \vec{H}\right] = \frac{1}{en_e c}\left[\vec{j} \times \vec{H}\right] + \frac{1}{\sigma}\,\vec{j} \;. \qquad (17)$$

(n_e = electron density; σ = ohmic conductivity). This equation has the same value in the filament as in the surrounding corona. No appreciable current can flow obliquely to the magnetic field in the corona since it doesn't result in a force which is able to maintain the equilibrium with the resulting Lorentz force. We further assume that the corona is in a state of rest ($v = o$), and because of eq. (17) $\vec{E}$ can possess no component oblique to the magnetic field. Because our symmetry assumptions this means that the components of $\vec{E}$ parallel to the plane of the filament vanish, because of the rotational freedom of $\vec{E}$ (stationary state!) this is valid also inside the filament. Thus eq. (17) applied to the interior of the filament is sufficient in order to determine the velocity of the matter from the subsequent current following from:

$$(92) \qquad \frac{1}{c}\left[\vec{j} \times \vec{H}\right] = \rho\,\vec{g} \qquad (18)$$

in order to determine the velocity of the matter from the subsequent current.

For the vertical component it follows:

$$(93) \qquad v_z = \frac{\rho\, g\, c^2}{\sigma H^2}$$

with $\rho = 10^{-14}$ g cm^{-3}, $g = 10^{4.4}$ cm sec^{-2}, $H_x = H_y = H = 1$ gauss and $\sigma = 10^{13}$ el. stat. cgs - units it is

$$(94) \qquad v_z \approx 10^{-1.6} \text{ cm/sec} .$$

It follows for the horizontal component that

$$(95) \qquad v_y = \frac{\rho\, g\, c}{e\, n_e\, H} \approx 10^{0.5} \text{ cm/sec} .$$

Both velocity components are so small that they are completely eliminated from observation in so far as they become annihilated, in any case, with variations from the presupposed stationary case.

8. Comments on the Three Dimensional Problem

Real filaments have only a finite length though it be large and also the magnetic fields do not possess the properties required up to now

($\partial\vec{H}/\partial y = o$; $H_y = o$). We must, therefore, go to a treatment of the three dimensional case. Because of the great mathematical difficulties, which arise, we must limit ourselves to a more qualitative consideration. The current which flows in a filament held in an equilibrium position is essentially directed horizontally. If one assumes that the current leaves the filament only at those places where the filament touches the solar surface (fig. 6) then at the locations A and B of the filament where the current possesses a vertical component, horizontal forces have to show up which want to extend the filament in its longitudinal direction.

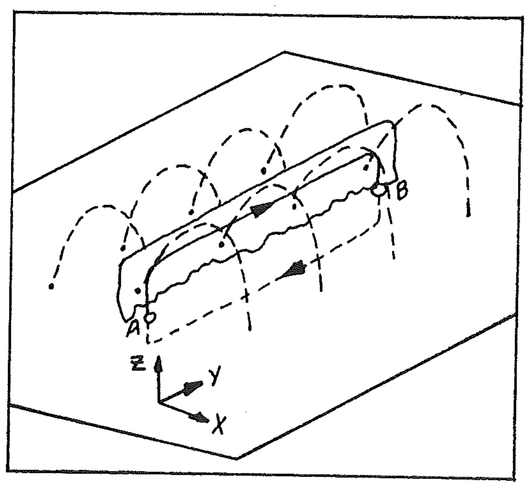

Fig. 6. First case of the possible current courses in a filament. The irregular surrounded figure is the filament. — — — — field lines, ——▶—— current above the solar surface, — — ▶ — current below the solar surface.

Since these forces are not compensated by an opposing force each filament must extend its formation in the longitudinal direction until its ends arrive at the positions at which the magnetic field strength vanishes and the current can flow into the solar surface without the action of a force. This is consistent with the observations since the lengths of the filaments of equal order of magnitude are as the diameter of Babcock's "field-patches."

The direction of flow of the current drawn in fig. 6 doesn't describe a single possibility by which the filament current can terminate. The current can also leave the filament and can drain into the solar surface by way of the corona (fig. 7). However, in the corona the magnetic field $\vec{H}$ governs, where $\vec{H}$ is composed of the undisturbed field and the field of the current. Therefore no forces act on the current flowing in the corona except those that are not compensated for by other forces, so the current must flow along the field lines of $\vec{H}$. The field external to the filament is then no longer a potential field, but on the contrary a solution of the

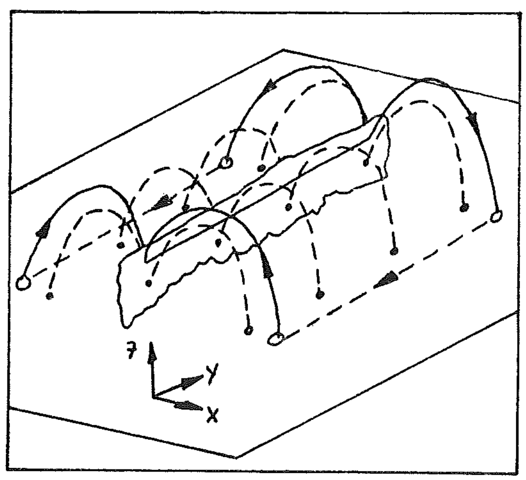

Fig. 7. Second kind of possible current flows in a filament. (Same key to symbols as figure 6.)

non-linear equation

$$(96) \qquad \vec{j} \times \vec{H} = o \quad .$$

If one eliminated j with the aid of the Maxwell equation (2) one obtains

$$(97) \qquad \vec{H} \times (\nabla \times \vec{H}) = o \; .$$

One then has the problem of the "force free" magnetic fields in the neighborhood of the filament. Solutions of this kind, which also play an important part in cosmical hydrodynamics, have, up to now, only been found in a few special cases (S. Lundquist 1950, R. Lüst and A. Schlüter 1954, A. Schlüter 1957) in which the geometric stipulations differed essentially from those which are layed down here. It is, therefore, not possible to make allowances for this with simple mathematical means, as it was with the case of the stability conditions. It must, however, change to our conditions (7) for filaments with sufficiently small mass, since the current outside the filament will be small and doesn't change the reaction of the magnetic field to the stability conditions. It seems to be apparent that the three dimensional magnetic field belonging to two magnetic dipoles (which lie on two points of the solar surface in such a way that their main direction coincides with the z direction) is capable of supporting a filament stably.

If the case reproduced in figure 6 is realized it will make our preceding considerations plausible. If the case represented in figure 7 is realized then it follows only for filaments of sufficiently small total mass. It is uncertain what happens if the total mass of the filaments in the case of figure 7 become so large that the streams flowing outside of the filament disturb the field strength. In this case we are not allowed to say more. We must have more information on criterion (7) and must allow for a mechanism of the field lines penetrating under the solar surface.

9. Estimates

Our solution for the microstructure of a filament permits a few comparisons with observation. From (15) it follows that the thickness of a filament is something like four times the homogeneous thickness $h = \frac{RT}{g\mu}$. If one sets $H_z^\infty / H_x \approx 1$ it means that the bend of the field lines of the filament is around 90° (one allows the field lines to run into the filament so that d becomes correspondingly larger). If one assumes ($\mu = 1$, $T = 3700°$K) then $d \approx 500$ km. The observer estimates the thickness of a filament to be at most 5000 km. One must, therefore, consider that the observed values which depend on the measurements of the projection of a filament onto the solar disk can only represent an upper limit for the actual filament thickness since it depends on the assumption that the filament possesses no irregularities in its geometric form as a function of height. On the other hand, the theory produces the larger value for d if $H_z^\infty / H_x < 1$. It appears to be the theory most compatible with the observations.

From (16) one obtains for the density ρ in the middle of the filament

$$\rho_c = \frac{1}{8\pi} (H_z^\infty)^2 \frac{\mu}{RT} \quad . \tag{98}$$

One takes $H_z^\infty = 1$ gauss, corresponding to Babcock's measurement, and one uses $\mu = 1$, $T = 3700°$ and obtains for ρ_c the value $10^{-12.9}$ g cm^{-3} corresponding to a number of $10^{10.9}$ hydrogen atoms per cm^3. This result agrees with the densities derivable from the observations (cp. Unsöld, 1955, p. 700).

We bring a few estimates, in the following, which relate to the balance of matter in a filament. One of the filaments supported by a magnetic field has a constant loss of matter by "ambipolar diffusion." Although the matter is ionized, neutral atoms are formed by recombination. These neutral atoms are no longer contained by the magnetic field and they can sink until they become ionized again. This produces a stream of matter in the direction of gravity. It is a noteworthy condition for the correctness of the theory that a filament doesn't lose a substantial part of its matter in a short time by this ambipolar diffusion without obtaining sufficient supplementary matter. One accepts that the diffusion velocity v_D, with which the neutral atoms fall onto the sun, is small in comparison to the thermal velocity v_{th}. Thus if we assume pure hydrogen v_D becomes

$$(99) \qquad v_D = \frac{2 \rho g}{n_n n_i m_p Q v_{th}} = \frac{2 g}{n_i Q v_{th}} , \quad \left(v_{th} = \frac{2}{\pi} \sqrt{\frac{2\pi kT}{\mu}} \right)$$

(n_n number of neutral atoms per cm^3, n_i the number of ions per cm^3, m_p proton mass, Q is the excitation cross section of ions and neutral atoms). The number of atoms per cm^2 leaving the base of the filament is then

$$(100) \qquad N_{out} = n_n v_D = \frac{2g}{Q v_{th}} \frac{n_n}{n_i} , \quad [cm^{-2} sec^{-1}] .$$

This mass loss of the filament opposes the gain of mass falling in from coronal atoms. The number of atoms entering per cm^2 of filament surface is

$$(101) \qquad N_{in} = \frac{n_K V_{thK}}{4} \quad [cm^{-2} sec^{-1}]$$

(n_K density of atoms in the neighborhood of the filament, v_{thK} is the thermal velocity in the corona). All of the atoms entering remain in the filament since the free path is of the order of magnitude 10^{+3} cm which is smaller than the thickness of the filament. One takes $n_k = 10^{8.6}$ cm^{-3}, $V_{thK} = 10^{7.3}$ cm sec^{-1}, a suitable coronal temperature of 1.6×10^{6}°K, and then $N_{in} = 10^{15.3}$ cm^{-2} sec^{-1}. One sets $T \approx 10^{4}$°K in the filament with $\mu = 1$, $Q = 10^{-15}$ cm^2 and then $N_{out} = 10^{13.5} \frac{n_n}{n_i}$ cm^{-2} sec^{-1}. With $\frac{n_n}{n_i} = 10^{-1}$

(Unsold 1955, p. 691) it follows that $N_{out} = 10^{12.5}$ cm^{-2} sec^{-1}. From this follows that by far less matter leaves a unit area of basal surface of the filament than is introduced through a unit area of lateral surface. From the geometry of the filament it follows that a magnetic field can contain a filament in spite of the ambipolar diffusion and on the contrary the filament receives a strong magnetic influx from the corona. This excess of mass admittance is decreased if one takes into account the thinning of the corona in the neighborhood of the filament in the calculation. Furthermore, the mass loss, which is hard to estimate, will occur on the surface by evaporation making the problem more difficult. If never the less the mass increase becomes too large the filament will begin to sink under until it goes into the solar surface and will decrease the mass to be carried. Perhaps it is the appearing of the remarkable "saugflusse" on the bottom edge of the filament uniting this. It is thought that the material balance of the filament is tightly coupled with the energy balance. If for example we allow an energy dissipation proportional to the volume (some by radiation, turbulence or mechanical waves) in the interior of the filament, then it heats up the filament and the thinning of the atoms on the surface grows. The filament would then possess the density for the stationary case, which would be important in order to reach such a strong heating that the evaporation on the solar surface would be in equilibrium with the incident particles. The question about the energy balance of a filament is, however, so tightly linked with the question on the heating mechanism of the corona, that one can scarcely successfully attack the first before finding the solution of the latter problem.

10. Filaments and Observed Magnetic Fields

The preceding theory can only be proved by observation. To a certain extent we have already done this when we had found the theoretical results of density, length and thickness of the filament in reasonable agreement with practical experience. Also the basic assumption of this theory up to now is open to confirmation. This is we can now find if the magnetic field runs horizontally in the filament. Since 1953 the magnetic field on the solar disk has been measured daily by H. D. and H. W. Babcock. One observes the field component along the line of sight (longitudinal Zeeman-Effect) and in the middle of the disk they observed practically the vertical field component. In the middle section of the solar disk filaments should only appear where the measured field strength disappears. When one of the authors (A. Schlüter) was a guest of the Mount Wilson and Palomar Observatories in August 1954 it was tried to prove this relation by putting the original material at the disposal of A. Schlüter and Dr. H. W. Babcock. Unfortunately, the number of filaments in a narrow extent in these regions in this period are few and also the localization of the measured field strengths on the sun were not sufficiently complete for this special purpose so that a definite decision was not possible. It was determined not to be in disagreement with the theory and the impression was won that the theoretical prediction was fulfilled in all cases. It is intended to extend this proof and then to report fully on it.*

One can obtain indirect indications between filaments and magnetic fields if he assumes that the geometric structure of the magnetic field is

* Note by the proofreader: In the meantime one was able to use the improved recording method of H. W. Babcock and the previously stated correlation between the disappearance of H_z and the position of the filament was confirmed.

sufficiently simple. This means that the surfaces on which the field is horizontal stands somewhat perpendicular to the field lines. Then the filament should be orientated approximately perpendicular to the field. In general the filaments are arranged, in lower latitudes, in a somewhat north-south direction - approximately perpendicular to the field regions ("field patches"). In higher latitudes - in the neighborhood of the polar caps, they both appear equal and polarized in the opposite sense to each other - the field is presumably arranged in a north-south direction and the orientation of the shifting filament appears to conform nearly with the parallel of latitude.

11. Remarks on the "disparitions brusques" and on the theory of corpuscular radiation

We will add a few remarks here which go beyond the realm of a theory of stationary prominences.

It is a well known phenomenon in the normal life of a filament to suddenly disappear and there exists a significant probability that the filament will appear anew at the place of its old location. This can be arranged in the proposed theory: we have already emphasized that a realistic theory of the stationary prominence essentially presents a stability problem. Perhaps only relatively small changes in the outer ("undisturbed") magnetic field or in the mass distribution of the filament cause them to violate the stability conditions. But then, the considerable energy stored by the deformation of the magnetic field is available and can be changed into potential or kinetic energy of the matter; or looked at in another way the field lines bent downwards from the filaments have the tendency to stretch out again and in this way to hurl matter up. After the filament has so exploded the matter falls back along the field lines onto the solar surface and the magnetic field again retains the essential form of the previous undisturbed field. That is the magnetic field lines again become running horizontally where the filament had been found and when the stability conditions are again fulfilled a new filament can then form on the old location.

In order to maintain a base over which the stored energy in the deformed magnetic field over a base we consider the case of two monopole fields (section 2) in which the energy difference ΔE dy between the field deformed by a filament and the undisturbed field for a segment dy of the y axis. It is

$$\Delta E\,dy = \frac{m^2\,dy}{8\pi}\left[\iint(\vec{H}^o + \vec{H}^f)^2\,dx\,dz - \iint(\vec{H}^o)^2\,dx\,dz\right] \tag{102}$$

$$= \frac{m^2\,dy}{4\pi}\int_o^\infty\int_{-\infty}^{o}\left\{\left[\frac{x+1}{r_1^2} - \frac{x-\alpha}{r_\alpha^2}\right]^2 + z^2\left[\frac{1}{r_1^2} - \frac{1}{r_\alpha^2}\right]^2 \right. \tag{103}$$

$$\left. + \left[\frac{x+1}{r_1^2} - \frac{x-1}{r_2^2}\right]^2 - z^2\left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right]^2\right\} dx\,dz$$

$$= \frac{m^2\,dy}{8}\left(2\ln\left|\frac{1+\alpha}{2}\right| - \ln\alpha\right), \quad (o < \alpha < 1), \tag{104}$$

where m is the pole strength of the monopole and r_1, r_2, α, r_α have the same meaning as in section 2.

For the total mass of a filament of thickness dy one obtains

$$M\,dy = dy\int_o^\infty f\,dz = -\frac{dy\,m^2}{2\pi g}\int_o^\infty\left(\frac{1}{r_1^2} + \frac{\alpha}{r_\alpha^2}\right)_{x=o} z\left(\frac{1}{r_1^2} - \frac{1}{r_\alpha^2}\right)_{x=o} dz \tag{105}$$

$$= \frac{m^2\,dy}{2\pi g}\left(\frac{1-\alpha}{2\alpha} - \frac{1}{\alpha+1}\ln\alpha\right). \tag{106}$$

We now ask about the height s up to which one can raise the total mass Mdy of the strip under consideration with energy $\triangle E$ dy. It is

$$s = \frac{\Delta E}{Mg} = \frac{\pi}{4} \frac{2 \ln \left| \frac{1 + \alpha}{2} \right| - \ln \alpha}{\frac{1 - \alpha}{2\alpha} - \frac{1}{\alpha + 1} \ln \alpha} \quad . \tag{107}$$

Here s measured the half interval of both monopoles. For $\alpha = 1/2$ one obtains $s \approx 0.1$. One considers violating the stability of the filament by insignificant changes of the magnetic field against disturbances in the x-direction so that a large portion of the filamentary matter becomes orientated laterally along the field lines which flow down onto the solar surface. The rest of the filamentary material can then be hurled with half of the stored energy in the field to heights greater than s.

We now compare this "catapult" mechanism of the "disparitions brusques" with the earlier presented "melon seed" theory (A. Schluter 1954) of the acceleration of corpuscular radiation. With the earlier theory the matter moves between the field lines and a force appears where the undisturbed field is homogeneous and the energy comes from the thermal energy of the accelerated material. With the catapult mechanism on the other hand the field lines are placed in the matter. Therefore, one can move the matter relative to the magnetic field only along the field lines and one doesn't see how this mechanism can lead to corpuscular radiation which finally leaves the sun and the sun's magnetic field. Besides this the arrangement limits the supply of standing energy and it cannot be restored by the heating mechanism of the melon seed theory which will change mainly the warmth.

The preceding investigation was begun when one of the authors (A. Schlüter) was the holder of a scholarship for further education abroad as a guest of the Princeton University Observatory and the Mount Wilson and Palomar Observatories. It was finished when the other author (R. Kippenhahn) was the guest of the astrophysical section of the Max-Planck Institut for Physics. A Schlüter would like to thank the Dr.'s M. Schwarzschild and H. W. Babcock for many discussions and essential assistance. We both thank Dr. Prof. Biermann for his interest and his advice.

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